

Deep Dive 2

Differentiation and its Applications

This deep dive assignment focuses on limits, continuity, and derivatives.

Assessment

This problem set has two sections:

- Deep Dive Problems: This section contains novel problems that explore the concepts introduced in class. Each problem will be graded on the tagged LOs.
- Reflection Questions: This section prompts you to reflect on your learning and highlight your LO applications. This section will be graded on and the LOs you highlight.

Collaboration

Math is a lot more fun with friends. We encourage you to collaborate and to discuss the problem sets with each other. However, it is key that you are able to apply the skills and concepts you learn in class and explain your approach on your own. For this reason, you should not share any written work with each other outside of class and are required to write up all solutions independently. You should also acknowledge the peers you collaborated with on your submission and cite any external resources you consult.

Submission

You may choose to type your solutions using L^AT_EX, Google Documents, or other software. You may also write on a graphic tablet and submit a PDF of your work. Hand-written solutions on paper are also acceptable, so long as they are scanned/photographed clearly and are fully legible. There exist scanning apps that will allow you to take photos of pages and create a PDF from them (Office Lens, for example). Low scores will be assigned for illegible work and/or images of poor quality. All graphs should be included in your PDF file, with a relevant caption.

Deep Dives

1. *Least Squares* (**#differentiation**, **#theoreticaltools**) Adapted from Stewart's "Calculus"

Suppose that a scientist has reason to believe that two quantities x and y are related linearly, that is $y = mx + b$, at least approximately, for some values of m and b . The scientist performs an experiment and collects data in the form of points (x_1, y_1) , (x_2, y_2) , $\dots$, (x_n, y_n) , and then plots these points. The points don't lie exactly on a straight line, so the scientist wants to find constants m and b so that the line $y = mx + b$ "fits" the points as well as possible.

Let $d_i = y_i - (mx_i + b)$ be the vertical deviation of the point (x_i, y_i) from the line. The **method of least squares** determines m and b so as to minimize $\sum_{i=1}^n d_i^2$, the sum of the squares of these vertical deviations. Show that, according to this method, the line of best fit is obtained when m and b satisfy the following system of equations:

$$\begin{aligned} m \sum_{i=1}^n x_i + bn &= \sum_{i=1}^n y_i \\ m \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i &= \sum_{i=1}^n x_i y_i \end{aligned}$$

2. *Breaking Lagrange* (**#differentiation**, **#computationaltools**) Consider the problem of maximizing the function $f(x, y) = 3x + 4y$ subject to the constraint $\sqrt{x} + \sqrt{y} = 7$.
 - (a) What are the critical points you obtain using Lagrange Multipliers? Classify them as local/global maxima or minima.
 - (b) Create a contour plot to illustrate the method of Lagrange multipliers the problem by graphing the constraint equation and several level curves of f . Use your diagram to identify any other critical points.
 - (c) Were there any extreme values not identified by the method of Lagrange Multipliers? Why?

Reflection Questions

1. Give two examples of applying LOs to solve a problem. How did you apply the LO? Was the application successful? If yes, why? If not, how could you improve your application?
2. Write up a detailed solution of one of the breakout problems from any class session between Related Rates and Lagrange Multipliers II. Your intended audience should be a ICS111 peer who struggled with the material. Make sure to explain your steps and highlight any use of computational tools. This is helpful for the technical interview.